Chemical Potential, Mixtures and Reactions

Elements of Biophysics

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Folding and
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Phase and Transitions

- A substance is described as having a form of matter termed a phase when the substance is uniform in chemical composition and physical state.

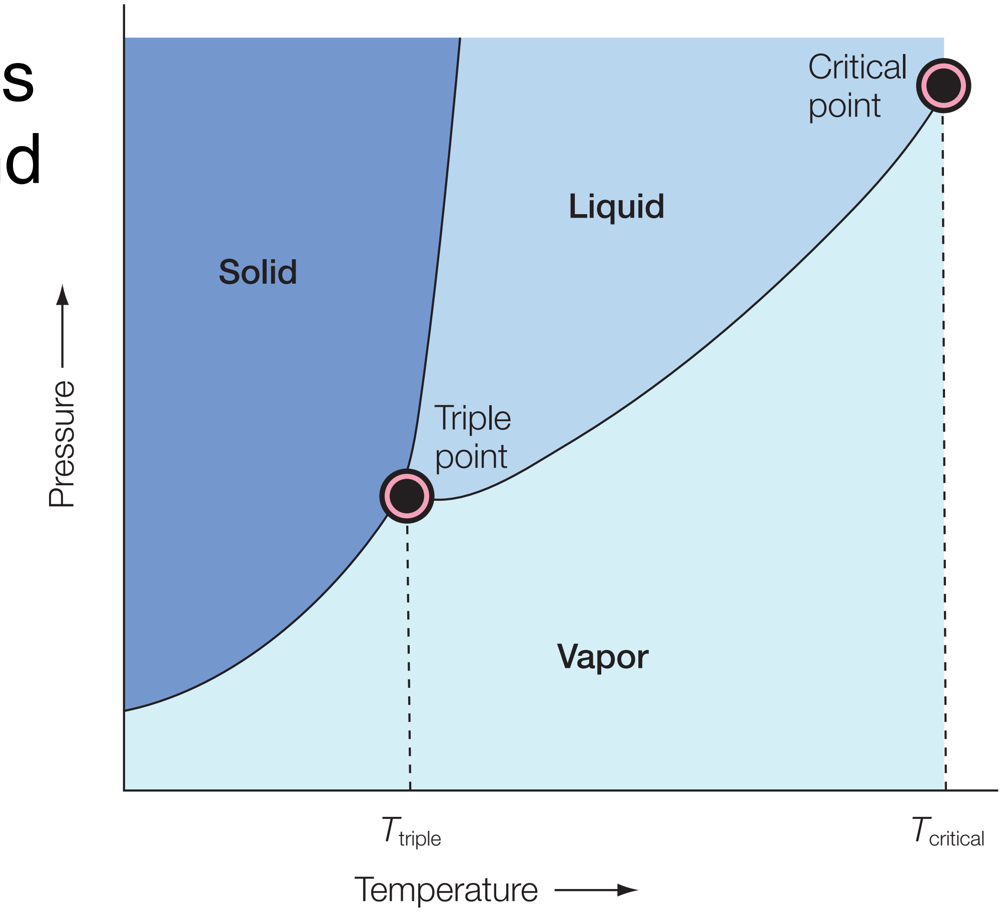
- Substance may exist in solid, liquid, or gases phases that can co-exist

- Substances may make a phase transition, which is a spontaneous conversion from one phase into another phase. Phase transitions occur at characteristic temperatures and pressures.

- At the transition temperature, the two phases are in equilibrium as the Gibbs energy is at a minimum value.

Phase Diagram

- A line between two states represents conditions at which both states can exist in equilibrium and is called a phase boundary.



- The phase diagram has a special point called the triple point which is located at the intersection of the three phase boundaries

- When the temperature is at or above the critical point, the liquid and vapor states are no longer distinctive

Chemical potential

- The chemical potential is equal to the Gibbs energy per mole of substance, or equivalently the molar Gibbs energy for a pure substance.

$$\mu = \frac{G}{n} \text{ or equivalently } G = n\mu$$

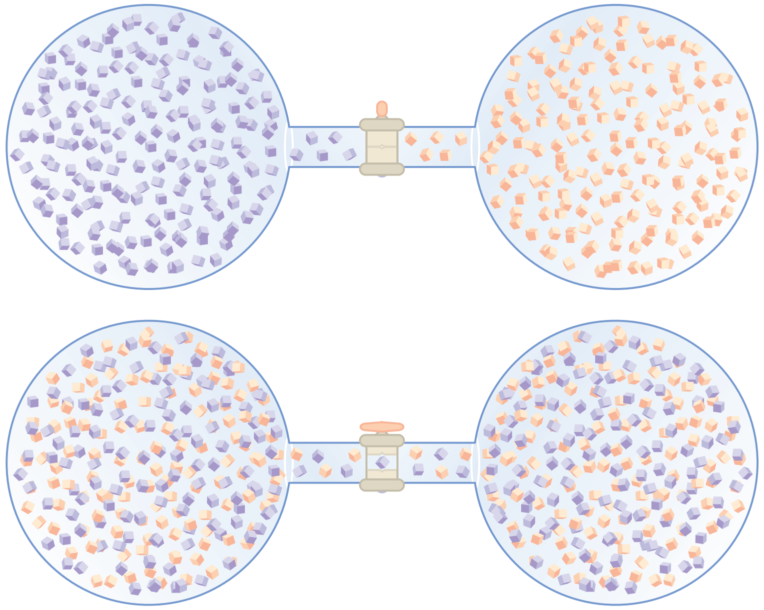
- For mixtures of multiple substances these contributions are additive. The total Gibbs energy for a mixture is equal to the sum of the individual partial Gibbs energies for each component of the mixture. For example, with two substances

$$G = n_{\mathrm{A}}\mu_{\mathrm{A}} + n_{\mathrm{B}}\mu_{\mathrm{B}}$$

Mixture

Consider a container that has two different ideal gases A and B in separate containers, the variation of Gibbs energy at constant T for a single gas is

$$G = n_A\mu_A + n_B\mu_B \qquad \mu_A - \mu_A^i = \frac{G_A}{n_A} = RT\ ln\frac{P_j}{P_i}$$



If at the final state the gasses have pressures P_A and P_B respectively with $P{=}P_A{+}P_B$

$$\Delta G_{mix} = G_f - G_i = n_A\left[\cancel{\mu_A(P_0)} + RTln\frac{P_A}{P_0}\right] + n_B\left[\cancel{\mu_B(P_0)} + RTln\frac{P_B}{P_0}\right]$$

$$-n_A\left[\cancel{\mu_A(P_0)} + RTln\frac{P}{P_0}\right] - n_B\left[\cancel{\mu_B(P_0)} + RTln\frac{P}{P_0}\right]$$

$$= n_ART\left[ln\frac{P_A}{P_0} - ln\frac{P}{P_0}\right] + n_BRT\left[ln\frac{P_B}{P_0} - ln\frac{P}{P_0}\right]$$

$$= n_ART\ ln\frac{P_A}{P} + n_BRT\ ln\frac{P_B}{P} = nRT\left[X_AlnX_A + X_B\ lnX_B\right]$$

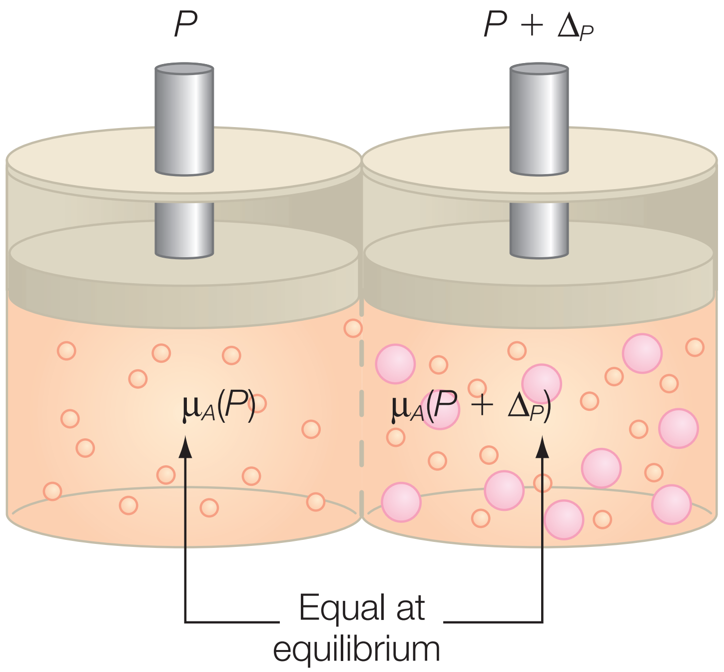
with $X_A = \frac{n_A}{n} = \frac{P_A}{P} \quad X_B = \frac{n_B}{n} = \frac{P_B}{P} \quad n = n_A + n_B$

Osmosis

Osmosis is the spontaneous movement of a pure solvent into a solution across a semipermeable membrane that allows the solvent, but not the solute, to pass through.

Since the side with the solute has a lower chemical potential, the solvent will migrate across the membrane towards the side which higher concentration.

The osmotic pressure is defined as the pressure that must be applied to the side with the solute to prevent solvent transfer



$$\mu_A(P + \Delta P) - \mu_A(P) = RTlnX_A \qquad A = solvent \quad B = solute$$

At equilibrium, the difference in chemical pressure is balanced by the difference in pressure multiplied by the volume:

$$(\Delta P)V = -n_A RTlnX_A = -n_A RTln(1 - X_B) \approx n_A RTX_B = n_A RT\frac{n_B}{n} \quad \Rightarrow$$

$$\Delta P = M_B RT \quad when \quad n_A \approx n \quad M = molarity$$

Gibbs Energy Minimum

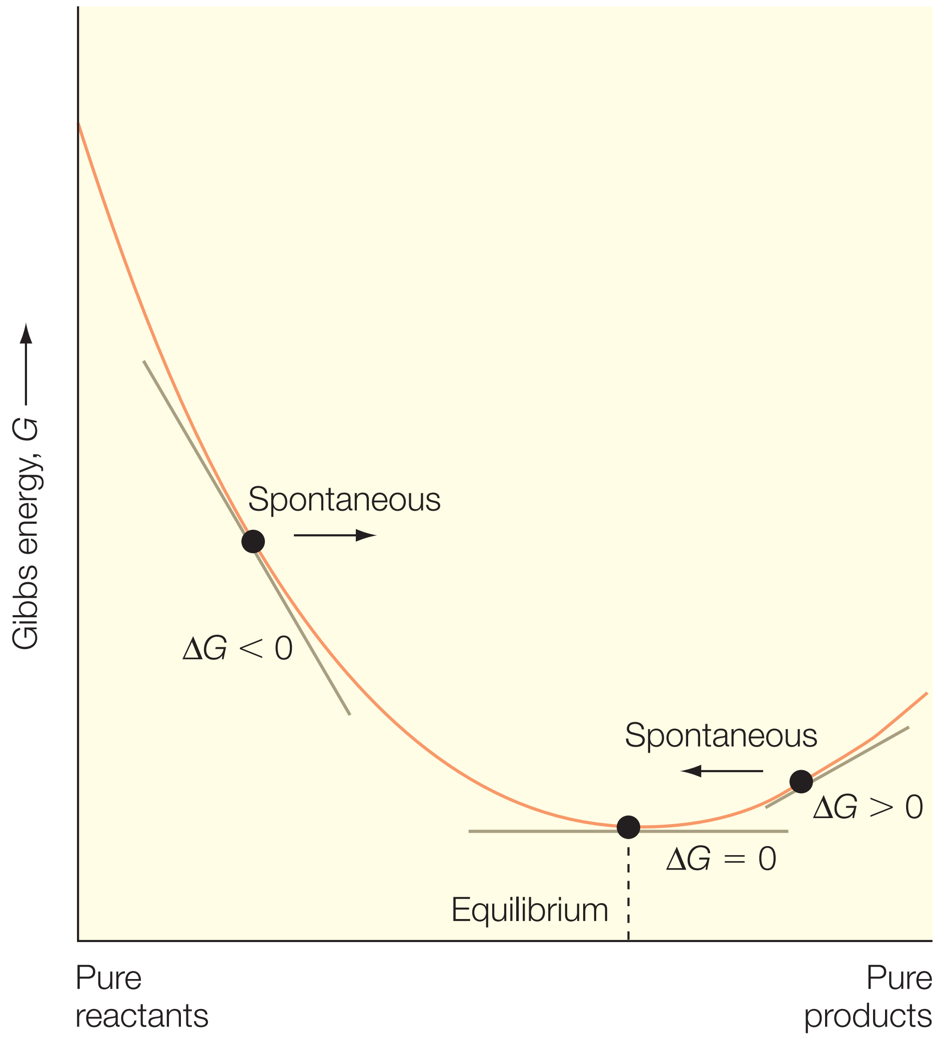
- After a reaction has started, it will eventually reach an equilibrium that depends upon the Gibbs energy difference, ΔG.

- At the equilibrium, the value of G is at the minimum and moving away from the equilibrium is energetically unfavorable.

- The slope of the free-energy dependence shows the direction in which the reaction will proceed.

- The Gibbs energy difference can be written as in logarithmic terms of the equilibrium constant (K_{eq})

$$\Delta G^{o} = -RT \ln K_{eq}$$



Gibbs Energy and Equilibrium

Using previous equation used to model a mixture we can define the variation to the Gibbs Energy of the reaction $A \rightarrow B$ as:

$$(\Delta G)_{rec} = \mu_B - \mu_A = \mu_B^0 + RTln\frac{P_B}{P} - \mu_A^0 - RTln\frac{P_A}{P}$$

$$(\Delta G)_{rec} = \mu_B^0 - \mu_A^0 + RTln\frac{P_B}{P_A}$$

$$(\Delta G)_{rec} = (\Delta G)_{rec}^0 + RTln\frac{P_B}{P_A}$$

The minimum of the Gibbs energy will occur when its variation is zero and the reaction will neither go forwards nor backwards, because the slope is zero. At this equilibrium point:

$$(\Delta G)_{rec} \;=\; 0 \;=\; (\Delta G)_{rec}^0 + RTln\frac{P_B}{P_A} \;\Rightarrow\; (\Delta G)_{rec}^0 = -RTln\frac{P_B}{P_A}$$

if $K_{eq} = \frac{P_B}{P_A} \;\Rightarrow\; K_{eq} = e^{-\frac{(\Delta G)_{rec}^0}{RT}}$

Equilibrium Constant

Considering a general reaction with two reactants and two products

$$aA \;+ bB \rightarrow cC + dD \qquad\qquad K_{eq} = \frac{a_C^c a_D^d}{a_A^a a_B^b} \quad a_i = activity = \gamma c_i$$

This equation can be obtained using the procedure described above

$$\begin{aligned}(\Delta G)_{rec} = c\mu_C + d\mu_D - a\mu_A - b\mu_B = d(\mu_D^0 + RTlna_D) + c(\mu_C^0 + RTlna_C)\\ -a(\mu_A^0 + RTlna_A) - b(\mu_B^0 + RTlna_B)\\ = (d\mu_D^0 + c\mu_C^0 - a\mu_A^0 - b\mu_B^0)+\\ RT(dlna_D + cna_C - alna_A - blna_B)\end{aligned}$$

At the equilibrium $(\Delta G)_{rec} = 0$ considering

$$(\Delta G)_{rec}^0 = (d\mu_D^0 + c\mu_C^0 - a\mu_A^0 - b\mu_B^0) \quad and \quad \gamma = 1 \qquad \Rightarrow$$

$$(\Delta G)_{rec}^0 = -RT(dlna_D + clna_C - alna_A - blna_B) = -RTln\frac{c_C^c c_D^d}{c_A^a c_B^b} = -RTlnK_{eq}$$

Acid-Base Equilibria (pK$_W$)

Water molecule can serve as donor or acceptor of a proton. Accordingly the following reaction is valid

$2H_2O \leftrightarrow H_3O^+ + OH^-$ and K$_w$ is $K_W = \dfrac{a_{\mathrm{OH}^-} a_{\mathrm{H_3O^+}}}{a_{\mathrm{H_2O}} a_{\mathrm{H_2O}}} = a_{\mathrm{OH}^-} a_{\mathrm{H_3O^+}}$ $a_{H_2O} = 1$

Since the energy of a reaction is related to the natural logarithm of the equilibrium constant it is convenient to calculate $pK = -\,logK$. Given this definition

$$\mathrm{p}K_W = -\log(K_W) = -\log(a_{\mathrm{OH}^-} a_{\mathrm{H_3O^+}}) = -\log(a_{\mathrm{OH}^-}) - \log(a_{\mathrm{H_3O^+}})$$

$$\mathrm{pH} = -\log(a_{\mathrm{H_3O^+}})$$

$$\mathrm{pOH} = -\log(a_{\mathrm{OH}^-})$$

$$\mathrm{p}K_W = \mathrm{pH} + \mathrm{pOH}$$

for water at 25 ^{o}C the K_w is 1.008 x 10^{-14}

$$\mathrm{pH} + \mathrm{pOH} = 14 \quad \Rightarrow \quad \mathrm{pH} = \mathrm{pOH} = \frac{14}{2} = 7$$

Acid-Base Equilibria (pK$_A$)

Water is referred to as a weak acid or base since the equilibrium constant is small and the protons. When a weak acid is added to water

$HA + H_2O \leftrightarrow A^- + H_3O^+$ and K$_A$ is $K_A = \dfrac{[\mathrm{H_3O^+}][\mathrm{A^-}]}{[\mathrm{HA}]}$ $a_{H_2O} = 1$

thus:

$$pK_A = -\,logK_A = -\,log\frac{[H_3O^+][A^-]}{[HA]} = -\,log[H_3O^+] - log[A^-] + log[HA]$$

$pK_A = -\,2log[H_3O^+] + log[HA] = 2pH + log[HA]$ when $[H_3O^+] = [A^-]$

$$\Rightarrow \quad pH = \frac{1}{2}pK_A - \frac{1}{2}log[HA]$$

Titration curve

Using the previous equation we can plot the titration curve which represent the variation of pH at different volumes of strong acid or base

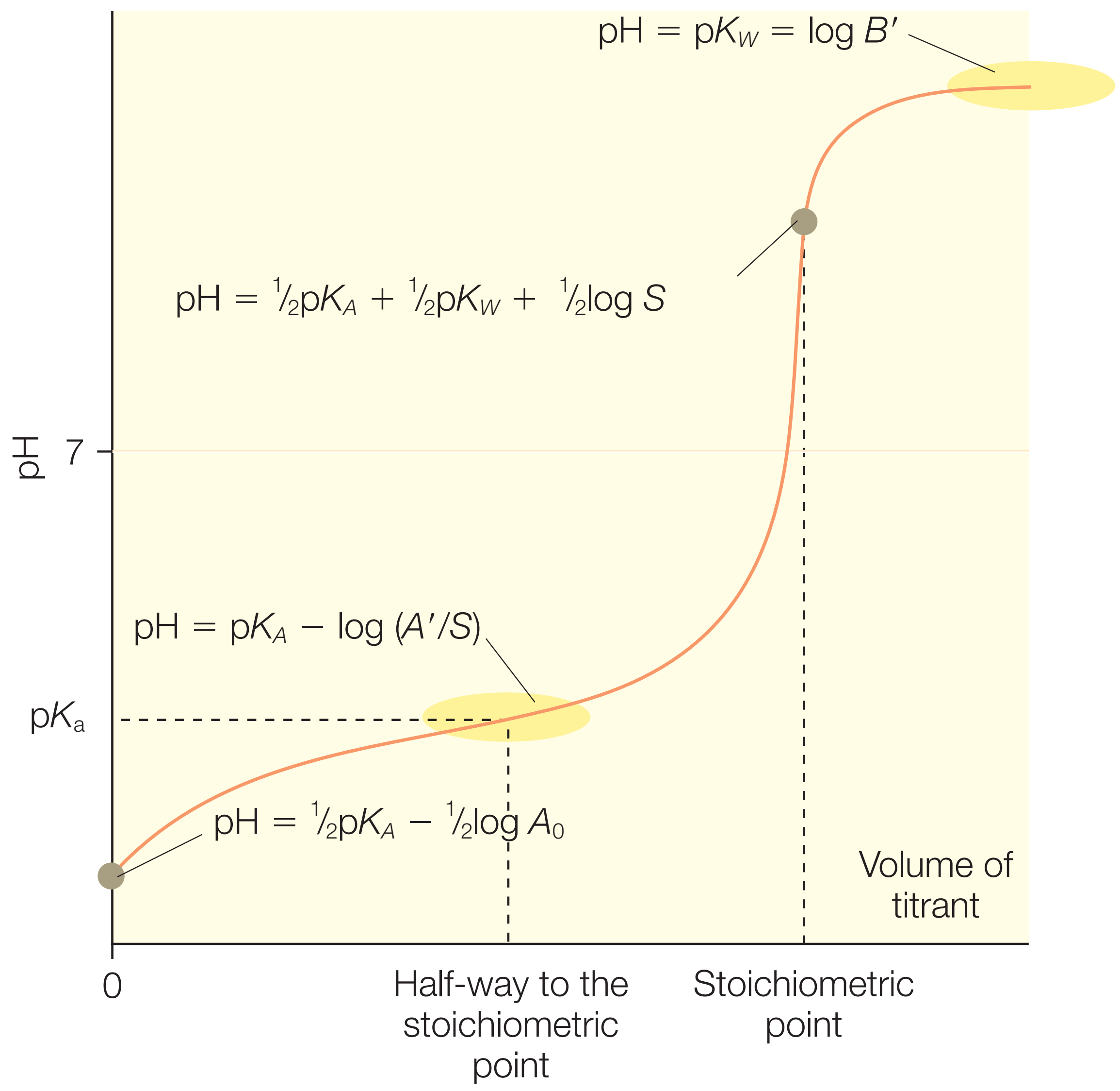
$$pH = \frac{1}{2}pK_A - \frac{1}{2}log[HA]$$

which is equivalent to

$$pH = pK_A + log\frac{[A^-]}{[HA]}$$

Half-way to the stoichiometry

$pH = pK_A$ when $[HA] = [A^-]$



Stoichiometry Point

The stoichiometric point is when enough strong base has been added to convert all of the weak acid HA to the conjugate weak-base form A⁻.

$$A^- + H_2O \leftrightarrow HA + OH^-$$

$$K_B = \frac{[\mathrm{HA}][\mathrm{OH^-}]}{[\mathrm{A^-}]} = \frac{[\mathrm{HA}]}{[\mathrm{H_3O^+}][\mathrm{A^-}]}[\mathrm{H_3O^+}][\mathrm{OH^-}] = \frac{1}{K_A}K_W$$

$$[\mathrm{HA}] \approx [\mathrm{OH^-}]$$

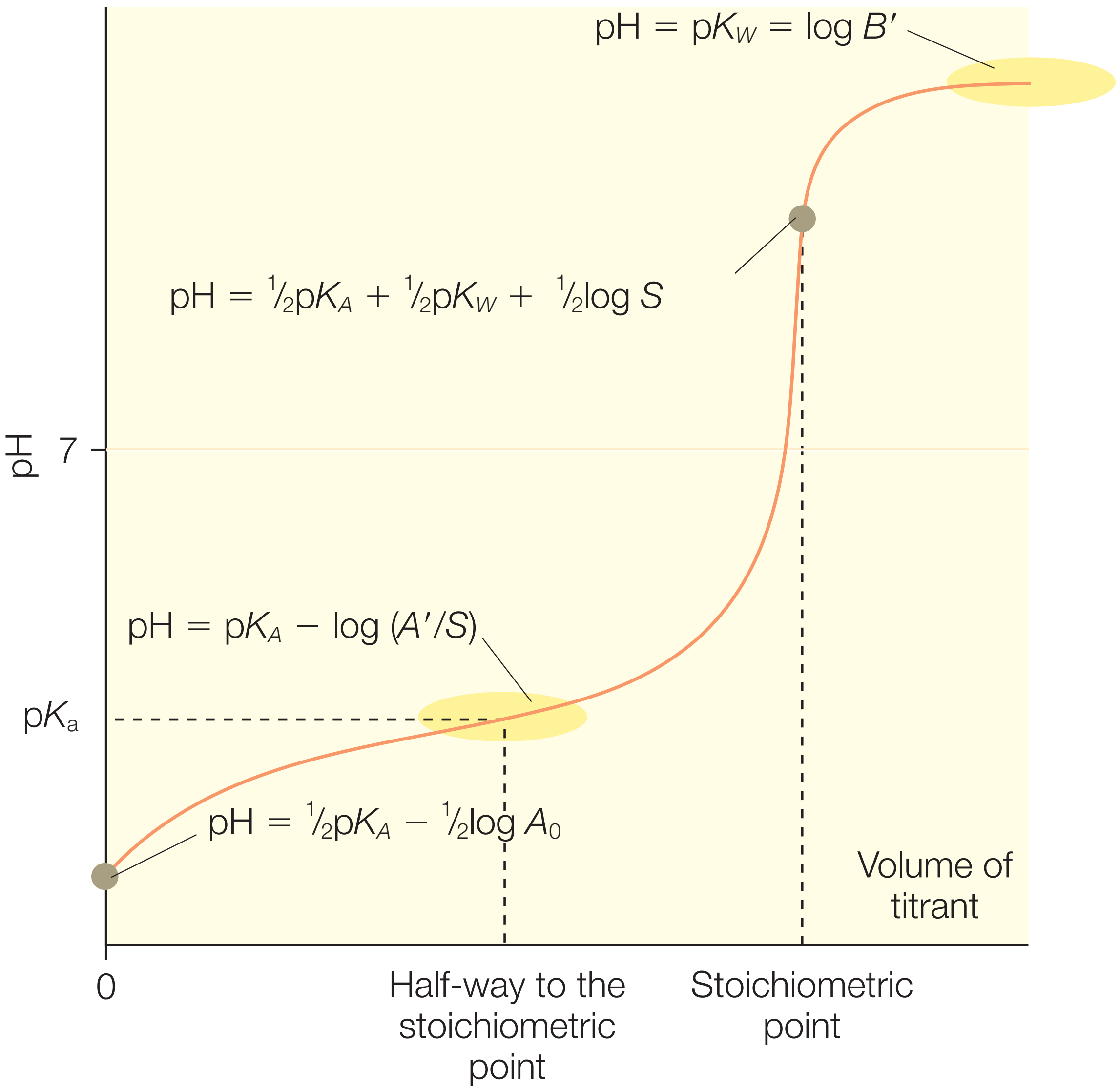
$$\frac{K_W}{K_A} = \frac{[\mathrm{HA}][\mathrm{OH^-}]}{[\mathrm{A^-}]} = \frac{[\mathrm{OH^-}]^2}{[\mathrm{A^-}]}$$

$$\frac{1}{[\mathrm{A^-}]}\frac{[\mathrm{OH^-}]^2[\mathrm{H_3O^+}]^2}{[\mathrm{H_3O^+}]^2} = \frac{1}{[\mathrm{A^-}]}\frac{K_W^2}{[\mathrm{H_3O^+}]^2}$$

$$\log\frac{K_W}{K_A} = \log\frac{1}{[\mathrm{A^-}]}\frac{K_W^2}{[\mathrm{H_3O^+}]^2}$$

$$-\mathrm{p}K_W + \mathrm{p}K_A = -\log[\mathrm{A^-}] - 2\mathrm{p}K_W + 2\mathrm{pH} \text{ or}$$

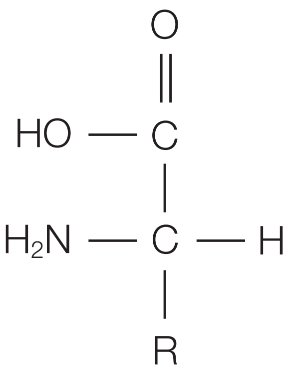
$$\mathrm{pH} = \frac{1}{2}\mathrm{p}K_A + \frac{1}{2}\mathrm{p}K_W + \frac{1}{2}\log[\mathrm{A^-}]$$



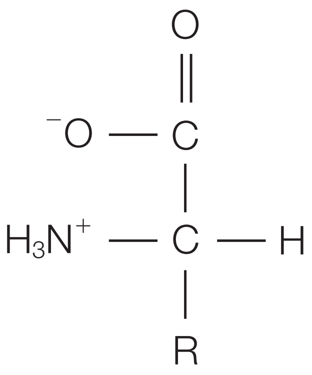
Amino Acids

When an amino acid is dissolved in water, it can exchange a proton with water, acting as either a weak base or a weak acid

$$NH_3^+ \leftrightarrow NH_2 + H^+$$
$$COO^- \leftrightarrow COOH$$

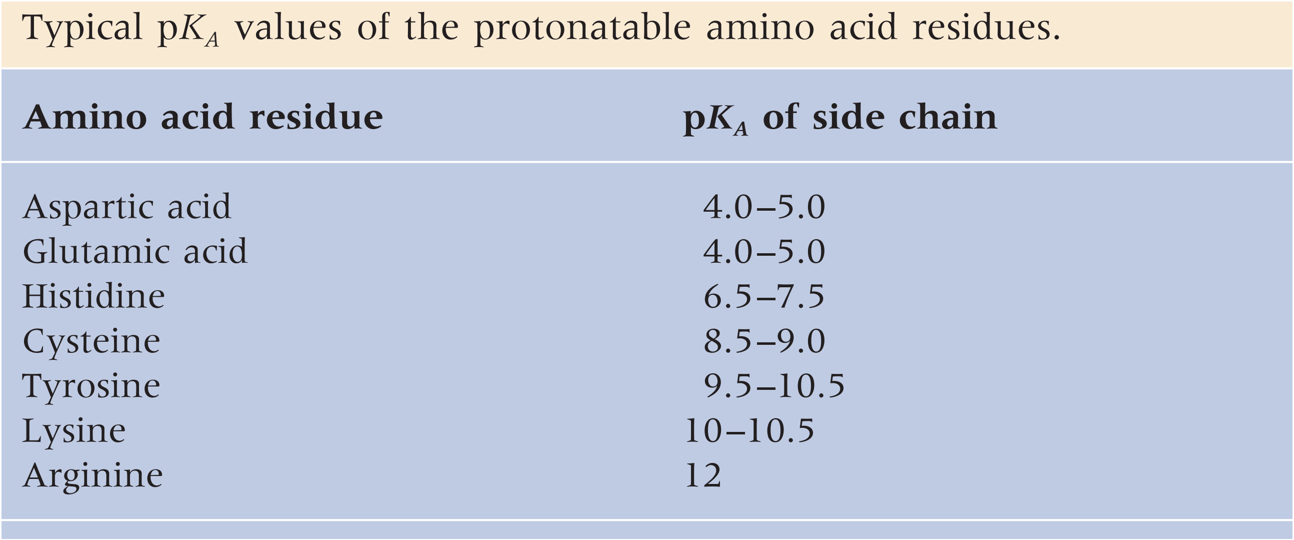


Nonionic form



Zwitterionic form

Thus, there are at least two *pK_A* values, of around 2.35 and 9.60, associated with every amino acid. Of the 20 common amino acid residues, only seven have side chains that can be protonated, and the *pK* values range from 4.0 to 12



Typical pK_A values of the protonatable amino acid residues.

Amino acid residue	**pK_A of side chain**
Aspartic acid	4.0–5.0
Glutamic acid	4.0–5.0
Histidine	6.5–7.5
Cysteine	8.5–9.0
Tyrosine	9.5–10.5
Lysine	10–10.5
Arginine	12